

**CONTROL OF A 2 D.O.F DIRECT DRIVE ROBOT ARM USING  
INTEGRAL SLIDING MODE CONTROL**

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**A project report submitted in partial fulfilment of the  
requirements for a award of the degree of  
Master of Engineering ( Electrical-Mechatronics and Automatic Control)**

**Faculty of Electrical Engineering  
Universiti Teknologi Malaysia**

**OCTOBER, 2004**

To my dearest father, mother and family for their encouragement and blessing

To my beloved be expecting wife for her support and caring ... ..

## **ACKNOWLEDGEMENT**

First of all, I am greatly indebted to ALLAH SWT on His blessing to make this project successful.

I would like to express my gratitude to honourable Associate Professor Dr. Mohamad Noh Ahmad, my supervisor of Master's project. During the research, he helped me a lot especially in guiding to use Matlab/Simulink software. Then, during the discussion session, he tried to give me encouragement and assistance which finally leads me to the completion of this project.

Finally, I would like to dedicate my gratitude to my parents, my family, my lovely wife and my best friends who helped me directly or indirectly in the completion of this project. Their encouragement and guidance mean a lot to me. Their sharing and experience foster my belief in overcoming every obstacle encountered in this project.

Guidance, co-operation and encouragement from all people above are appreciated by me in sincere. Although I cannot repay the kindness from them, I would like to wish them to be well and happy always.

I am grateful to Kolej Universiti Teknologi Tun Hussein Onn ( KUiTTHO), (my employer) for supporting me in the form of a scholarship and study leave.

## **ABSTRACT**

High accuracy trajectory tracking is a very challenging topic in direct drive robot control. This is due to the nonlinearities and input couplings present in the dynamics of the robot arm. This thesis is concerned with the problems of modelling and control of a 2 degree of freedom direct drive arm. The research work is undertaken in the following five developmental stages; Firstly, the complete mathematical model of a 2 DOF direct drive robot arm including the dynamics of the brushless DC motors actuators in the state variable form is to be developed. In the second stage, the state variable model is to be decomposed into an uncertain model. Then, the Integral Sliding Mode Controller is applied to the robot arm. In the forth stage, perform the simulation. This is done through the simulation on the digital computer using MATLAB/SIMULINK as the platform. Lastly, the performance of Integral Sliding Mode Controller is to be compared with an Independent Joint Linear Control.

## **ABSTRAK**

. Penjejak trajektori yang berketepatan tinggi merupakan satu topik yang mencabar dalam kawalan robot pacuan terus. Ini adalah disebabkan oleh ketaklelurusan dan gandingan masukan yang wujud di dalam dinamik lengan robot. Tesis ini membincangkan mengenai masalah dalam permodelan dan kawalan lengan robot yang mempunyai 2 darjah kebebasan. Kajian ini melibatkan lima peringkat seperti berikut; Pertama, pembangunan model matematik 2 DOF lengan robot pacuan terus yang lengkap merangkumi dinamik pemacu motor DC tanpa berus dalam bentuk pembolehubah keadaan. Di peringkat kedua, model pembolehubah keadaan akan dipisahkan ke model yang tidak tetap. Kemudian, kawalan ragam gelincir kamiran diguna pakai dalam lengan robot ini. Peringkat kelima adalah membuat penyelakuan. Simulasi atau penyelakuan ini dijalankan menggunakan komputer digital dengan bantuan perisian MATLAB/SIMULINK. Akhir sekali, keupayaan di antara kawalan ragam gelincir kamiran dengan kawalan lurus bebas lipatan dibandingkan.

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## LIST OF SYMBOLS

<u>SYMBOL</u>	<u>DESCRIPTION</u>
<b>1.      UPPERCASE</b>	
$A(*,*)$	2N x 2N system matrix for the integrated direct drive robot arm
$A_{ac}$	2N x 2N system matrix for the augmented dynamic equation of the actuators
$\Delta A(*,*)$	matrix representing the uncertainties in the system matrix
$B(*,*)$	2N x N input matrix for the integrated direct drive robot arm
$B_{ac}$	2N x N input matrix for the augmented dynamic equation of the actuators
$\Delta B(*,*)$	matrix representing the uncertainties in the input matrix
$C$	N x 2N constant matrix of the PI sliding surface
$E(*)$	a continuous function related to $\Delta B(*,*)$
$F_i$	motor damping constant (kg.m <sup>2</sup> /s)
$H(*)$	a continuous function related to $\Delta A(*,*)$
$I^*$	brushless DC motor demand current
$I_{DC}$	brushless DC motor DC supply current
$J_i$	moment of inertia of the $i$ th motor (kg.m <sup>2</sup> )
$K_i$	linear feedback gain matrix for the $i$ th sub-system
$K_t$	motor torque constant (N.m/A)
$L_{s_i}$	stator winding inductance for $i$ th motor (H)
$N$	number of joints
$Q$	2N x N load distribution matrix for the augmented dynamic equation of the actuators

$R_{s_i}$	stator winding resistance for $i$ th motor ( $\Omega$ )
$\Re^N$	$N$ -dimensional real space
$S_\delta(t)$	a continuous function used to eliminate chattering
$T_{L_i}$	load torque for $i$ th motor (N.m)
$U(*)$	$N \times 1$ control input vector for a $N$ DOF robot arm
$X(*)$	$2N \times 1$ state vector for the integrated direct drive robot arm
$Z(*)$	$2N \times 1$ error state vector between the actual and the desired states of the overall system
$(*)^T$	transpose of $(*)$
$\ (*)^T\ $	Euclidean norm of $(*)$

## 2. LOWERCASE

$a_{ij}$	$ij$ th element of the integrated system matrix $A(*,*)$
$b_{ij}$	$ij$ th element of the integrated input matrix $B(*,*)$
$g$	gravity acceleration ( $m.s^2$ )
$l_i$	length of the $i$ th manipulator link (m)
$m_i$	mass of the $i$ th manipulator link (kg)
$t$	time (s)

## 3. GREEK SYMBOLS

$\alpha$	norm bound of continuous function $H(*)$
$\beta$	norm bound of continuous function $E(*)$
$\dot{\theta}$	joint displacement (rad)
$\ddot{\theta}$	joint velocity (rad/s)
$\ddot{\theta}$	joint acceleration ( $rad/s^2$ )
$\dot{\theta}_d$	desired joint angle (rad)

$\dot{\theta}_d$	desired joint velocity (rad/s)
$\ddot{\theta}_d$	desired joint acceleration (rad/s <sup>2</sup> )
$\sigma$	Integral sliding manifold
$\tau$	time interval for arm to travel from a given initial position to a final desired position (seconds)

## **LIST OF ABBREVIATIONS**

AC	Alternating Current
BLDCM	Brushless Direct Current Motor
DC	Direct Current
DOF	Degree of Freedom
IJC	Independent Joint Control
LHP	Left Half Plane
SMC	Sliding Mode Control
VSC	Variable Structure Control
VSS	Variable Structure System

## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 Overview**

High accuracy trajectory tracking is a very challenging topic in direct drive robot control. This is due to the nonlinearities and input couplings present in the dynamics of the arm. This thesis presents the modelling and control of a 2 DOF (degree of freedom) direct drive robot arm. Direct drive robot arm is mechanical arm in which the shafts of articulated joints are directly coupled to the rotors of motors with high torque. Therefore, it does not contain transmission mechanisms between motors and their load.

Serial kinematics chains have made up the majority of robot manipulator designs. Serious difficulties with serial mechanisms have proven difficult to overcome. High torques must be generated in the joints due to relatively long moment arms. Hydraulic actuators gave high force to weight ratios, but introduced maintenance and safety concerns. Electrical drives using gear trains, shafts, and couplings, could provide the necessary joint torques. However, these driving components introduced friction which reduced force control capabilities, and



backlash which reduced precision. Manipulator stiffness is also reduced by these drive components which are sometimes introduced to reduce the inertia of the links. Direct drive (DD) serial manipulators were introduced in the 1980's as a proposed solution to many of these problems [Asada and Kanade, 1983].

The direct drive joint consists of a pair of arm links, the motor, and the bearings. The motor is comprised of a stator and a rotor. The stator is housed in the case connected to a proximal link, and the rotor is directly coupled to the joint shaft, which is connected to the other arm link at a distal link. Thus the distal arm link is rotated directly by the torque exerted between the rotor and the stator, hence direct drive. Throughout this project, a two DOF direct-drive robot manipulator driven by Brushless DC Motors (BLDCM) is considered.

## **1.2 Objective**

The objectives of this research are as follows:

1. To formulate the complete mathematical dynamic model of the BLDCM driven direct drive revolute robot arm in state variable form. The complete model will be made available by integrating the dynamics of the 2 DOF direct drive robot arm with the BLDCM dynamics
2. To transform the integrated nonlinear dynamic model of the BLDCM driven direct drive robots into a set of nonlinear uncertain model comprising the nominal values and the bounded uncertainties. These structured uncertainties exist due to the limit of the angular positions, speeds, and accelerations.
3. To control the 2 DOF direct drive robot using Integral Sliding Mode Controller and to compare the performance between Integral SMC with other conventional controller.

## **1.3 Scope of Project**

The scopes of work for this project are

- The use of a 2 DOF Brushless DC motor driven Direct Drive Robot Arm as described in Reyes and Keyes(2001).
- Simulation work using MATLAB/Simulink as platform.

- The use of Integral Sliding Mode Controller as described in Ahmad et al.(2002).
- The comparison of the performance of Integral Sliding Mode Controller with Independent Joint Linear Control.

### **1.3 Research Methodology**

The research work is undertaken in the following five developmental stages:

- a) Development of the complete mathematical model of a 2 DOF direct drive robot arm including the dynamics of the Brushless DC Motors actuators in the state variable form.
- b) Decomposition of the complete model into an uncertain model.
- c) Utilize Integral Sliding Mode Controller as robot arm controller.
- d) Perform simulation of this controller in controlling a 2 DOF direct drive robot arm. This simulation work will be carried out on MATLAB platform with Simulink as it user interface.
- e) Compare of the performance of Integral Sliding Mode Controller with other controller such as Independent Joint Linear Control.

## **1.5 Advantages and Disadvantages of Direct Drive Robot Arm**

Positioning inaccuracy and tuning challenges are common in such systems due to transmission compliance and backlash. A direct drive rotary motor is simply a high torque permanent magnet motor that is directly coupled to the load. This design eliminates all mechanical transmission components such as gearboxes, belts, pulleys and couplings. Direct drive rotary systems offer a number of unique and significant advantages to the designer and user. Because a mechanical transmission requires constant maintenance and frequently causes unscheduled down time, direct drive rotary motors inherently increase machine reliability and reduce maintenance time and expense. By eliminating compliance in the mechanical transmission, the need for inertia matching of the motor to the load is eliminated, while position and velocity accuracy can be increased by up to 50 times [ Asada and Toumi, 1987].

Direct-drive technique eliminates the problems associated with gear backlash as well as reducing the friction significantly. Moreover, the mechanical construction is much stiffer than the conventional robot manipulator with gearing, wear and tear is not a problem, and the construction is more reliable and easy to maintain due to its simplicity. These features make the direct drive robots suitable for the high-speed application of industrial robot such as the laser cutting application.

A key factor for direct drive robot performance is the torque to mass ratio of the actuators. Direct drive robots had their own sets of difficulties. It cannot be controlled by simple controller due to the non linear coupled dynamic and input couplings.

The advantages and disadvantages of direct drive robot can be summarized as in Table 1.1.

**Table 1.1 : Advantages and Disadvantages of Direct Drive Robot**

<b>ADVANTAGES</b>	<b>DISADVANTAGES</b>
<ul style="list-style-type: none"><li>▪ No gear backlash and lower friction.</li><li>▪ Simpler construction.</li><li>▪ No wear and tear</li><li>▪ Fast and accurate</li></ul>	<ul style="list-style-type: none"><li>▪ Very non linear coupled dynamic</li><li>▪ Consist of uncertainties</li><li>▪ Input couplings</li><li>▪ Cannot be controlled by simple controller</li></ul>

## **1.6 Literature Review**

The purpose of manipulator control is to maintain the dynamic response of a manipulator in accordance with pre-specified objectives. The dynamic performance of a manipulator directly depends on the efficiency of the control algorithms and the dynamic model of the manipulator. Most of current industrial approaches to the robot arm control design treat each joint of the manipulator as a simple linear servomechanism with simple controller like Independent Joint Control (IJC), proportional plus derivative (PD), or proportional plus integral plus derivative (PID) controllers. In this approach, the nonlinear, coupled and time-varying dynamics of the mechanical part of the robot manipulator system have usually been completely ignored, or assumed as disturbances. However, when the links are moving simultaneously and at high speed, the nonlinear coupling effects and the interaction forces between the manipulator links may decrease the performance of the overall system and increase the tracking error. The disturbances and uncertainties such as variable payload in a task cycle may also reduce the tracking quality of the robot manipulator system [Osman, 1991].

Since the early days of the robot, robot control system has become an active

research area. Various advanced and sophisticated control strategies have been proposed by numerous researchers for controlling the robot manipulator such that the system is stable as well as the motion of the manipulator arm is maintained along the prescribed trajectory. The structures of these controllers can be grouped into three categories, the centralized, decentralized, and multilevel hierarchical structures. Many strategies have been developed for the centralized control schemes for improving the control of the nonlinear and coupled time varying robot manipulator. These include among others, the Computed Torque techniques [Craig, 1989], Adaptive control strategies [Ortega and Spong, 1988] and the Variable Structure Control approaches [Young, 1978].

Variable Structure Control (VSC) with sliding mode control was first proposed and elaborated in the early 1950's in the Soviet Union by Emelyanov and several co-researchers. Since 1980, two developments have greatly enhanced the attention given to VSC systems. The first is the existence of a general VSC design method for complex systems. The second is a full recognition of the property of perfect robustness of a VSC system with respect to system perturbation and disturbances [John and James, 1993].

A sliding mode will exist for a system if in the vicinity of the switching surface, the state vector is directed towards the surface. Filippov's method is one possible technique for determining the system motion in a sliding mode, but a more straight forward technique easily applicable to multi-input systems is the method of equivalent control, as proposed by Utkin and Drazenovic [DeCarlo, et. al 1988].

The motivation for exploring uncertain systems is the fact that model identification of real world systems introduced parameter errors. Hence models contain uncertain parameters which are often known to lie within upper and lower bounds. A whole body of literature has risen in recent years concerned with the deterministic stabilization of systems having uncertain parameters lying within

known bounds. Such control strategies are based on the second method of Lyapunov. One has to take notice that, the plant uncertainties are required to lie in the image of input matrix  $B$  for all values of  $t$  and  $x$ . This requirement is the so-called matching condition [Gao and Hung, 1993]. The physical meaning of matching condition is that all modeling uncertainties and disturbances enter the system through the control channel [John and James, 1993].

Sliding mode techniques are one approach to solving control problems and are an area of increasing interest. In the formulation of any control problem there will typically be discrepancies between the actual plant and the mathematical model developed for controller design. This mismatch may be due to any number of factors and it is the engineer's role to ensure the required performance levels exist despite the existence of plant/model mismatches. This has led to the development of so-called robust control methods. However this approach is decreasing the order of the system dynamics, may produce undesirable result in certain application. Other alternative must be introduced to increase the order of the closed-loop dynamics [Ahmad, 2003].

To overcome the problem of reduced order dynamics, a variety of the sliding mode control known as the Integral Sliding Mode Control has been successfully applied in a variety of control. Different from the conventional SMC design approaches, the order of the motion equation in ISMC is equal to the order of the original system, rather than reduced by the number of dimension of the control input. The method does not require the transformation of the original plant into the canonical form. Moreover, by using this approach, the robustness of the system can be guaranteed throughout the entire response of the system starting from the initial time instance [Ahmad, 2003].

## **1.7 Layout of Thesis**

This thesis contained five chapters. Chapter 2 deals with the mathematical modelling of the direct drive robot arm. The formulation of the integrated dynamic model of this robot arm is presented. First, the state space representations of the actuator dynamics comprising of BLDCM motors are formulated. Then, the state space representations of the dynamic model of the mechanical linkage of the direct drive robot arm are established. Based on the actuator dynamics model, an integrated dynamic model of the robot arm is presented.

Chapter 3 presents the controller design using integral sliding mode control. The direct drive robot arm is treated as an uncertain system. Based on the allowable range of the position and velocity of the direct drive robot arms operation, the model comprising the nominal and bounded uncertain parts is computed. Then, a centralized control strategy for direct drive robot arm based on Integral Sliding Mode Control is described.

Chapter 4 discusses the simulation results. The performance of the Integral sliding mode controller is evaluated by simulation study using Matlab/Simulink. For the comparison purposes, the simulation study of Independent Joint Linear Control is also presented.

Chapter 5 summarizes the works undertaken. Recommendations for future work of this project are presented at the end of the chapter.



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